SHAH CLASSES

CULTIVATING SUCCESS SINCE 1998

Subject : Geometry **Total Marks : 40**

Class : X^{th} **Time : 2 Hr.**

Prelim Answer Paper - 1

Q.1 : A) Solve Multiple choice questions. 4

1) Ratio of areas of two similar tringles is 9:25. .... is the ratio of their corresponding sides.

Ans : b) 3:5

2) What is side and perimeter of square having diagonal $5\sqrt{2}$ cm.

Ans : b) 5 and 20 cm

3) Two circles of radii 5.5 cm and 4.2 cm touch each other externally. Find the distance between their centres.

Ans : a) 9.7

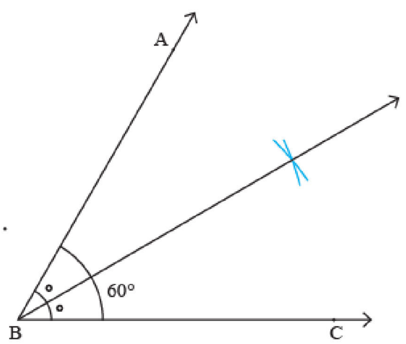
4) When we see at a higher level, from the horizontal line, angle formed is ______

Ans : a) angle of elevation.

B) Solve the following questions. 4

1) Construct $\angle ABC = 60^0$ and bisect it.

Ans :



2) $\triangle$ABC and $\triangle$DEF are equilateral triangles, A($\triangle$ABC) : A($\triangle$DEF) = 1 : 2. If AB = 4, then what is length of DE?

Ans : In $\triangle$ABC and $\triangle$DEF,

$\left.\begin{array}{l}\angle A \cong \angle D \\ \angle B \cong \angle E\end{array}\right\}$ [Each angle is of measure 60^0]

$\therefore \triangle ABC \sim \triangle DEF$ [AA test of similarity]

$\therefore \frac{A(\triangle ABC)}{A(\triangle DEF)} = \frac{AB^2}{DE^2}$ [Theroem of areas of Similar triangles]

$\therefore \frac{1}{2} = \frac{4^2}{DE^2}$

$\therefore DE^2 = 4^2 \times 2$

$\therefore DE = 4\sqrt{2}$

3) Identify, with reason, if the following is Pythagorean triplet. 3, 5, 4

Ans : (3, 5, 4)

$5^2 = 25$

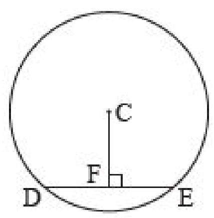
and $3^2 + 4^2 = 9 + 16$

$= 25$

$\therefore \quad 5^2 = 3^2 + 4^2$

$\therefore$ (3, 5, 4) is a Pythagorean triplets.

4) In the adjoining figure, seg DE is the chord of the circle with center C. Seg CF $\perp$ seg DE and DE = 16 cm, then find the length of DF.



Ans : The perpendicular drawn from centre to a chord bisects the chord.

Here seg CF $\perp$ seg DE (Given)

$\therefore$ $DF = \frac{1}{2} DE$

$\therefore$ $DF = \frac{16}{2}$

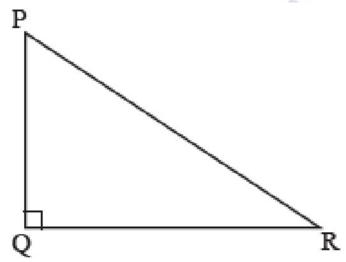
$\therefore$ DF = 8 cm .

Q.2 : A) Complete the following Activities. (Any two) 4

1) A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from the base of wall. Complete the given activity.

As show in fig.,

Ans : Suppose PR is the length of ladder = 10 m



At P - Window, At Q - base of wall, At R - foot of ladder

$\therefore$ PQ = 6m

$\therefore$ QR = ?

In $\angle PQR$, $m\angle PQR = 90^0$

$\therefore$ By Pythagoras theorem,

$= \pi(r_1 + r_2)1$ ... (1)

Here, PR = 10, PQ = $\boxed{8}$

$\therefore$ From equation (1)

$8^2 + QR^2 = 10^2$

$\therefore$ $QR^2 = \boxed{10^2} - \boxed{8^2}$

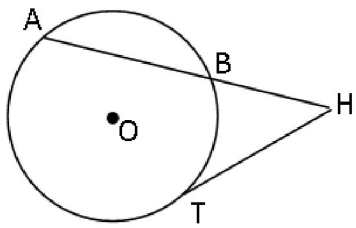
$\therefore$ $QR^2 = 100 - 64$

$\therefore$ $QR^2 = \boxed{36}$

$\therefore$ $QR^2 = 6$

$\therefore$ The distance of foot of the ladder form the base of wall is 6 m.

2) In the figure, T is the point of contact. HA = 9 cm and HB = 4 cm. Find HT.



Ans : HT is a tangent segment and HBA is the secant. By tangent secant property,

$HT^2 = \boxed{HB \times HA}$

$= 4 \times \boxed{9}$

$= \boxed{36}$

$\therefore$ $HT = \boxed{6}$ cm ... (Taking square root of both the sides)

3) Complete the following activity to find the coordinates of point P which divides seg AB in the ratio 3 : 1 where A (4, - 3) and B (8, 5)

Ans : $\therefore$ By section formula,

$\therefore$ $x = \frac{mx_2 + nx_1}{\boxed{m + n}}$,

$\therefore$ $x = \frac{3 \times 8 + 1 \times 4}{3 + 1}$,

$= \frac{24 + 4}{4}$

$\therefore$ $x = \frac{28}{4}$,

$\therefore$ x = 7

$y = \frac{\boxed{my_2} + \boxed{ny_1}}{m + n}$

$$y = \frac{3 \times 5 + 1 \times (-3)}{3+1}$$

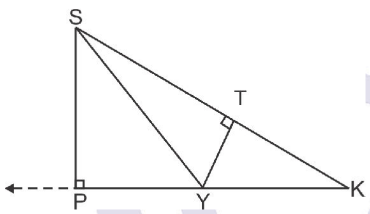
$$= \frac{\boxed{15} - 3}{4}$$

$$\therefore\ y = \frac{12}{4}$$

$y = 3$

B) Solve the following questions. (Any four) 8

1) In the figure, seg SP $\perp$ side YK and seg YT $\perp$ side SK.
If SP = 6, YK = 13, YT = 5 and TK = 12, then find A (ΔSYK) : A(ΔYTK).



Ans : The ratio of the areas of two triangles is equal to the ratio of the products of their bases and the corresponding heights.

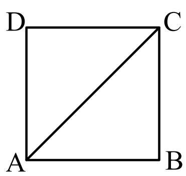
$$\therefore\ \frac{A(\Delta SYK)}{A(\Delta YTK)} = \frac{YK \times SP}{TK \times YT}$$

$$= \frac{13 \times 6}{12 \times 5} = \frac{13}{10}$$

$A(\Delta SYK) : A(\Delta YTK) = 13 : 10.$

2) Find the length of the hypotenuse of a square whose side is 16 cm.

Ans : Let $\square$ABCD is a square.



In right angled triangle ΔABC,

$AC^2 = AB^2 + BC^2$

... (by Pythagoras theorem)

$\therefore\ AC^2 = 16^2 + 16^2$

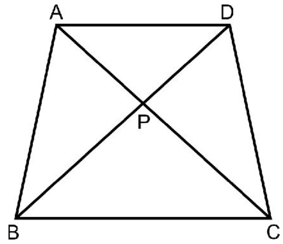
$\therefore\ AC^2 = 256 + 256$

$\therefore\ AC^2 = 512$

$\therefore\ AC = 16\sqrt{2}$

Diagonal AC= $16\sqrt{2}$

3)



In $\square$ABCD, seg AD $\parallel$ seg BC. Diagonal AC and diagonal BD intersect each other in point P.

Then show that $\frac{AP}{PD} = \frac{PC}{BP}$.

Ans : 1) Seg AD $\parallel$ seg BC and transversal AC ... [Given]

$\therefore\ \angle DAC \cong \angle BCA$ [Alternate angles theorem]

2) In ΔAPD and ΔCPB

$\angle APD \cong \angle CPB$ [Vertically opposite angles]

$\angle DAP \cong \angle BCP$ [from (1), A-P-C]

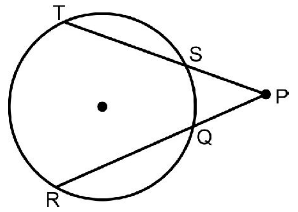
$\therefore\ \Delta APD \sim \Delta CPB$ [AA test]

3) $\frac{AP}{CP} = \frac{PD}{PB}$ ... [from (2), C.S.S.T]

4) $\therefore\ \frac{AP}{PD} = \frac{PC}{BP}$ ... [from (3), by alternendo]

i.e. $\frac{AP}{PD} = \frac{PC}{BP}$

4) In figure if PQ = 6, QR = 10, PS = 8 find TS.



Ans : Given : PQ = 6 units

QR = 10 units

PS = 8 units

To find : TS = ?

Solution :

PQ + QR = PR ... {P-Q-R}

6 + 10 = PR

$\therefore$ **PR = 16 units**

TP × SP = RP × QP

... {By property of intersecting chords outside the circle}

$\therefore$ TP × 8 = 16 × 6

$\therefore$ TP = 12 units

$\therefore$ TS + SP = TP ... {T-S-P}

$\therefore$ TS + 8 = 12

$\therefore$ TS = 12 - 8

= 4

$\therefore$ **TS = 4 units.**

5) If the slope of the line joining points (k, – 3) and (4, 5) is $\frac{1}{2}$, then find the value of k.

Ans : Let A (k, –3) $\equiv (x_1, y_1)$ and

B(4, 5) $\equiv (x_2\ y_2)$

The slope of line $AB = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{5-(-3)}{4-k} = \dfrac{5+3}{4-k} = \dfrac{8}{4-k}$

The slope is given to be $\frac{1}{2}$.

$\therefore \dfrac{8}{4-k} = \dfrac{1}{2}$ $\therefore$ 16 = 4 –k

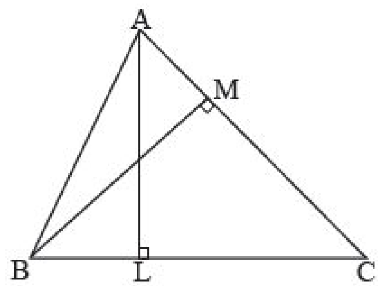
$\therefore$ k = 4 – 16

$\therefore$ k = – 12

The value of k is – 12.

Q.3 : A) Complete the following activity. (Any one) 3

1) In ΔABC, $AL \perp BC$ and $BM \perp AC$, B – L – C, A – M – C. Then show that $\Delta ALC \sim \Delta BMC$. If AL = 7, BM = 8, and BC = 12, then find AC. Complete the following activity.



Ans : In ΔALC and ΔBMC,

$\angle CLA = \angle CMB$ ... [Both 90^0]

$\angle ACL = \angle$ [BCM] ... (common angle)

$\therefore \Delta CLA \sim \Delta CMB$

... [By AA test of similarity]

$\therefore \dfrac{AL}{BM} = \dfrac{AC}{[BC]}$ ... [c.s.s.t]

$\therefore \dfrac{7}{8} = \dfrac{AC}{12}$

$\therefore$ AC = [10.5]

2) Radii of the top and the base of a frustum are 14 cm, 8 cm respectively. Its height is 8 cm.

Find its

i) curved surface area

ii) total surface area

iii) volume.

Ans : $r_1 = 14$ cm , $r_2 = 8$ cm, $h = 8$ cm

Slant height of the frustum = l

$= \boxed{\sqrt{h^2 + (r_1 - r_2)^2}}$ ---formula

$= \sqrt{8^2 + (14-8)^2}$

$= \sqrt{64 + 36}$

$= \boxed{10 \text{ cm}}$

Curved surface area of the frustum

$== \pi(r_1 + r_2)l$

$= 3.14 \times (14 + 8) \times 10$

$= \boxed{690.8 \text{ cm}^2}$

Total surface area of frustum

$= \boxed{\pi l (r_1 + r_2) + \pi r_1^2 + \pi r_2^2}$

$= 3.14 \times 10\ (14 + 8) + 3.14 \times 14^2 + 3.14 \times 8^2$

$= 690.8 + 615.44 + 200.96$

$= 690.8 + 816.4$

$= 1507.2$

Volume of the frustum =

$\boxed{\frac{1}{3} \pi h \left(r_1^2 + r_2^2 + r_1 \times r_2\right)}$

$= \frac{1}{3} \times 3.14 \times 8\ (14^2 + 8^2 + 14 \times 8)$

$= \boxed{3114.88 \text{ cm}^2}$

B) Solve the following questions. (Any two) 6

1) Find the ratio in which point P (k, 7) divides the segment joining A (8, 9) and B (1, 2). Also find k.

Ans : Let point P divides seg AB in the ratio m : n

Let A (8, 9) = (x_1, y_1),

B (1, 2) = (x_2, y_2) and

P (k, 7) = (x, y)

By section formula

$$y = \frac{my_2 + ny_1}{m+n}$$

$\therefore\ 7 = \frac{m(2) + n(9)}{m+n}$

$\therefore\ 7m + 7n = 2m + 9n$

$\therefore\ 7m - 2m = 9n - 7n$

$\therefore\ 5m = 2n$

$\therefore\ \frac{m}{n} = \frac{2}{5}$

Now $x = \frac{mx_2 + nx_1}{m+n}$

$\therefore\ k = \frac{2(1) + 5(8)}{2+5}$

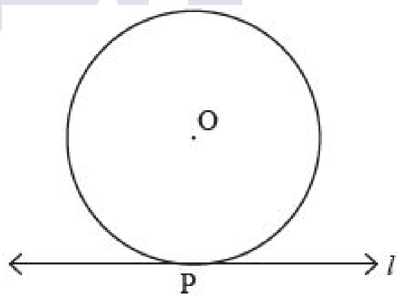
$\therefore\ k = \frac{2 + 40}{7}$

$\therefore\ k = \frac{42}{7}$

$\therefore\ k = 6$

$\therefore$ The ratio in which the point P divides seg AB is 2 : 5 and the value of k is 6.

2) Line l touches a circle with centre O at point P. If radius of the circle is 9 cm, answer the following.



i) What is d(O, P) = ? Why?

ii) If d(O, Q) = 8 cm, where does the point Q lie?

iii) If d(OR) = 15 cm, how many locations of point R are line on line l ? At what distance will each of them be from point P?

Ans : i) OP is the radius of the circle.

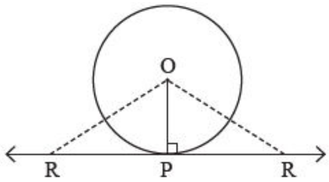
$\therefore$ d(O, P) = 9 cm (Given)

ii) d(O, Q) = 8 cm and radius = 9 cm

$\therefore$ Point Q lies indside the circle.

($\because$ d(O, Q) < r)

iii)



There can be two locations of point R on line *l*.

OP $\perp$ PR (Tangent radius theorem)

$\therefore$ In $\Delta OPR, \angle OPR = 90^0$

$\therefore$ $OR^2 = OP^2 + PR^2$ (Pythagoras theorem)

$\therefore$ $(15)^2 = 9^2 + PR^2$

$\therefore$ $PR^2 = 225 - 81$

$\therefore$ $PR^2 = 144$

$\therefore$ PR= 12 cm

$\therefore$ Point R is at a distance of 12 cm from point P on line l

3) If cos A + $\cos^2$ A = 1 then

$\sin^2$ A + $\sin^4$ A = ?

Ans : $\cos A + \cos^2 A = 1$ (Given)

$\therefore$ $1 - \cos^2 A = \cos A$ ... (1)

$\sin^2 A + \sin^4 A = \sin^2 A + \sin^2 A . \sin^2 A$

$= \sin^2 A + (1 - \cos^2 A)(1 - \cos^2 A)$

$= \sin^2 A + (\cos A)(\cos A)$

... [From (1)]

$= \sin^2 A + \cos^2 A$

$= 1$

$\therefore$ $\sin^2 A + \sin^4 A = 1$.

4) Do the following activity to draw tangents to the circle without using center of the circle.

a) Draw a circle with radius 3.5 cm and take any point C on it.

b) Draw chord CB and an inscribed angle CAB.

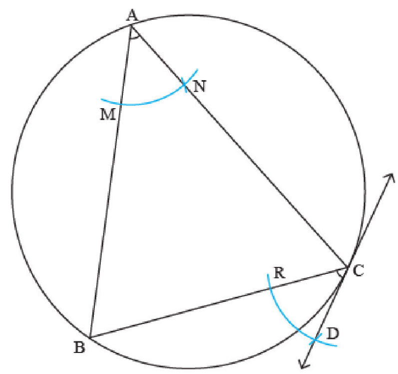
c) With the center A and any convenient radius draw an arc intersecting the sides of angle BAC in points M and N.

d) Using the same radius and center C, draw an arc intersecting the chord CB at point R.

Taking the radius equal to d(MN) and center R, draw an arc intersecting the arc

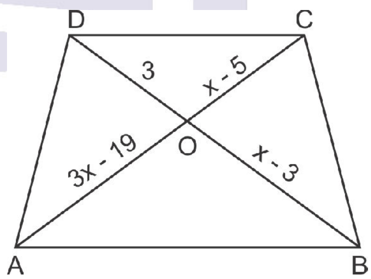
e) drawn in the previous step. Let D be the point of intersection of these arcs. Draw line CD. Line CD is the required tangent to the circle.

Ans :



Q.4 : Solve the following questions. (Any two) 8

1) In the figure, seg AB || seg DC. Using the information given find the value of x.



Ans : In $\square$ABCD,

seg AB || seg DC ... (given)

$\therefore$ $\angle DCA \cong \angle BAC$ ... (Alternate angles for transversal AC)

i.e. $\angle DCO \cong \angle BAO$ ... (A-O-C)

2) In ΔDOC and ΔBOA

a) $\angle DCO \cong \angle BAO$ ... [from (1)]

b) $\angle DOC \cong \angle BOA$ ... (vertically opposite angles)

c) $\Delta DOC \sim \Delta BOA$ ... (A - A test of similarity)

3) $\frac{DO}{BO} = \frac{CO}{AO}$ ... (c.s.s.t)

4) But DO = 3, BO = x – 3

CO = x – 5, AO = 3x – 19 ... (given)

5) $\therefore \frac{3}{x-3} = \frac{x-5}{3x-19}$ ... [from (3) and (4)]

$\therefore$ 3 (3x – 19) = (x – 5) (x – 3)

9x – 57 = x^2 – 8x + 15

$\therefore$ x^2 – 8x – 9x + 15 + 57 = 0

$\therefore$ x – 17x + 72 = 0

$\therefore$ (x – 8) (x – 9) = 0

either x – 8 = 0 or x – 9 = 0

$\therefore$ x = 8 or x = 9

The value of x = 8 or x = 9

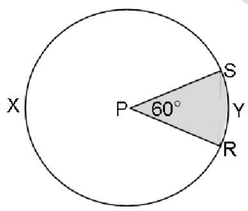
2) The radius of the circle is 7 cm and m (arc RYS) = 60° with the help of the fig, answer the following question:

i. Name the shaded protion.

ii. Find the area of the circle.

iii. Find A(P-RYS)

iv. Find A(P-RXS)



Ans : Given : radius (r) = 5 cm m(arc) = 60^0

i) The shaded portion is sector P-RYS

ii) Area of circle $= \pi r^2$

$$= \frac{22}{7} \times 7 \times 7 = 154 \text{ cm}^2$$

$\therefore$ Area of the circle is = 154 cm^2

iii) $\angle SPR = m(\text{arc } RYS)$ ...(Definition of minor arc)

$\therefore \angle SPR = 60^0$

$\therefore$ Measure of central angle (θ) = 60°

$\therefore$ Area of sector $= \frac{\theta}{360} \times \pi r^2$

$\therefore A(P-RYS) = \frac{60}{360} \times \frac{22}{7} \times 7 \times 7$

$$= \frac{1}{6} \times 154 = 25.66 \approx 25.67 \text{ cm}^2$$

$\therefore$ A(P-RYS) = 25.67 cm^2

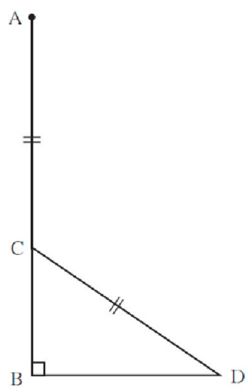
iv) Area of major sector = Area of circle - area of minor sector

$\therefore$ A(P-RXS) = area of circle – A(P-RYS)

$\therefore$ A(P-RXS) = 154 – 25.67 = 128.33 cm^2

3) A tree was broken due to storm. Its broken upper part was so inclined that its top touched the ground making an angle of 30°with the ground.The distance from the foot of the tree and the point where the top touched the ground was 10 metre. What was the height of the tree.

Ans :



As shown in figure, suppose AB is the tree. It was broken at ‘C’ and its top touched at ‘D’.

$\angle CDB = 30^0$, BD = 10 m, BC = x m

CA= CD = y m

In right angled ΔCDB,

$$\tan 30^0 = \frac{BC}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{10}$$

$$x = \frac{10}{\sqrt{3}}$$

$$y = \frac{20}{\sqrt{3}} \quad \text{(By } 30^0 - 60^0 - 90^0 \text{ theorem)}$$

$$x + y = \frac{10}{\sqrt{3}} + \frac{20}{\sqrt{3}}$$

$$= \frac{30}{\sqrt{3}}$$

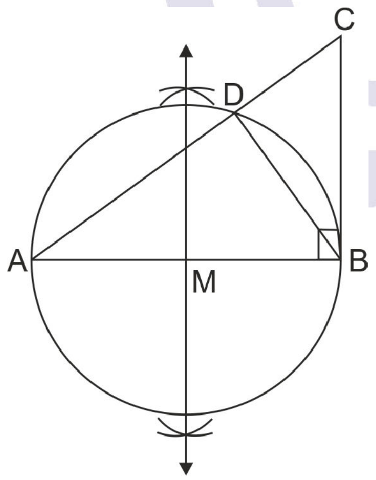
$$x + y = 10\sqrt{3}$$

$\therefore$ height of the tree was $10\sqrt{3}$ m.

Q.5 : Solve the following questions. (Any one) 3

1) Draw ΔABC such that, AB = 8, BC = 6 cm and $\angle B$ = 90°. Draw seg BD perpendicular to hypotenuse AC. Draw a circle passing through points B, D, A. Show that line CB is a tangent of the circle.

Ans :



Seg BD $\perp$ Seg AC

$\therefore$ ΔADB is a right angled triangle.

$\therefore$ Seg AB is a diameter of the circle passing through the points A, B and D

$\therefore$ Seg MB is a radius of the circle.

$\therefore$ MBC is a right angle ... (Given)

Thus BC is a tangent.

2) The surface area of a solid metallic sphere is 616cm^2 . It is melted and recast into smaller spheres of diameter 3.5cm. How many such spheres can be obtained?

Ans : Surface area of a metallic sphere = 616 cm^2

Since surface area of sphere $= 4\pi R^2$

$$\therefore \text{Radius (R)} = \sqrt{\frac{\text{Surface area}}{4\pi}}$$

$$= \sqrt{\frac{616 \times 7}{4 \times 22}} \text{cm}$$

$$= \sqrt{49}$$

$$= 7 \text{ cm}$$

$$\therefore \text{Volume of larger sphere} = \frac{4}{3}\pi R^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 \text{ cm}^3$$

$$= \frac{4312}{3} \text{cm}^3$$

Diameter of smaller sphere = 3.5cm

$$\text{Radius (r)} = \frac{3.5}{2} = \frac{7}{4} \text{cm}$$

$$\text{Volume of smaller spheres} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times \frac{7}{4}$$

$$= \frac{539}{24} \text{cm}^3$$

Number of smaller spheres

$$= \frac{4312}{3} \div \frac{539}{24}$$

$$= \frac{4312}{3} \times \frac{24}{539}$$

= 64 spheres.

* * *